Mathematical Model of DSM Tool For The Design And Modelling Of Fluid Transportation Pipeline

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ABSTRACT

In the oil and gas industry, there have been constant demands for more cost-effective and efficient tools to productive pipeline design. The task of pipeline design for effective implementation has only been done by performing expensive and time-consuming product development cycles which involve building the model, building a physical prototype of the design, testing the prototype in the field, evaluating the results of the field tests and modifying the design based on results of the field tests. This research interest, therefore, is to address these demands by coming up with a mathematical model using a domain specific modelling (DSM) tool solution equipped with type-systems and semantics that can simplify design and implementation of transmission pipeline structures. The metrics associated with the equations in the mathematical model presented in this paper with the accompanying figures indicate that the system can be used for front end engineering design (FEED), detailed engineering design and simulation of oil and gas flow on transmission pipelines. This DSM tool, which is a software suite conceived to be a robust system for fluid transportation pipeline design and modelling spans through the transient effect of temperature, pressure, stress, corrosion allowance for pipe, buckling effects for a successful flow assurance for piping systems, and operations for both offshore and onshore pipelines.

Keywords— *Domain Specific Modelling (DSM), front end engineering design (FEED), finite element method (FEM), computational fluid dynamics (CFD), and gas pipeline.*

1.0 INTRODUCTION

The Domain Specific Modelling (DSM) tool is a software suite conceived to be a robust system for fluid transportation pipeline design and modelling. As a modelling tool, it will work as a mechanical design automation application that allow engineers adequately sketch their idealized pipeline networks,

physical network components and experiment with features and dimensions and produce 3D models as well as detailed working drawings. On the other hand, it will also serve as a design tool that will be capable of performing analysis and simulation of both solid and fluid mechanics for all practical and potential flow scenarios as shown in **figure 1.**

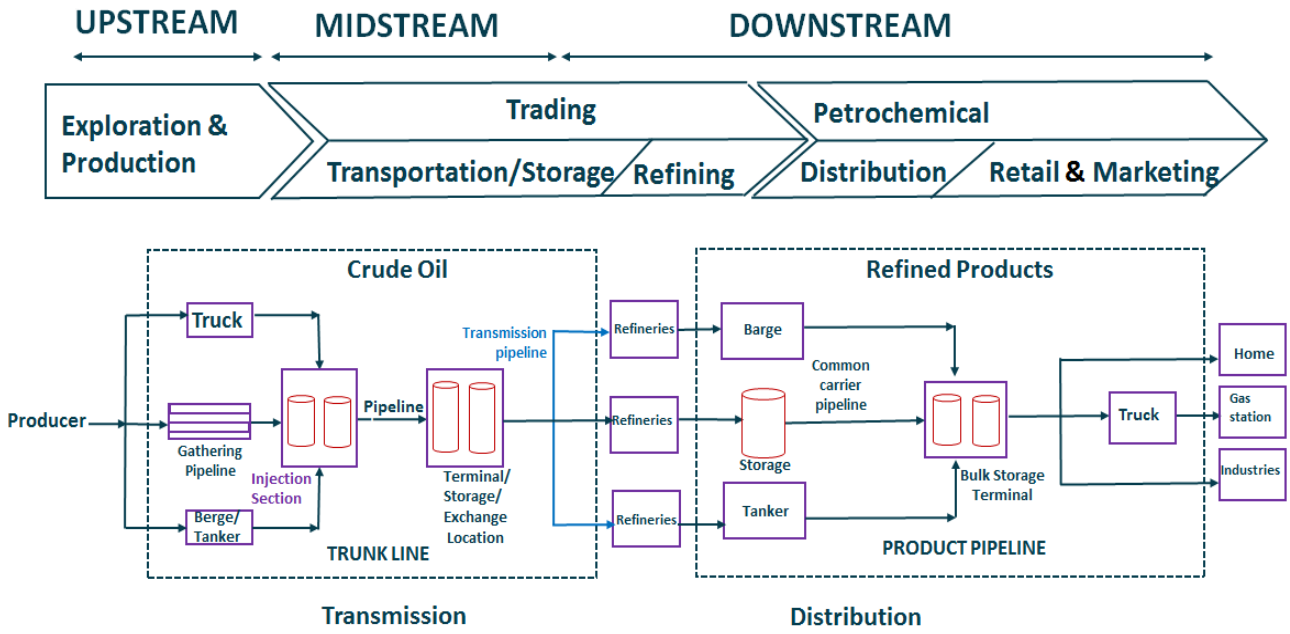

Figure 1. Transport Phenomena in the Oil and Gas Industry

The software do not focus only on the design of the physical components of a pipeline (pipe diameter, flanges, elbows etc.) but also tailored to serve all spheres of the oil and gas industrial pipeline application such as pipeline network simulation (computational solid mechanics), pipeline network flow simulation (computational fluid mechanics), and transient fluid flow in pipeline network design and modelling. Thus figure 1 adequately illustrates the piping and pipeline networks as it is applicable in the oil and gas industry.

A mathematical model is actually built to adequately actualize these potentials; this paper therefore is focused on the mathematical model in the system logic that drives the functionality of the DSM tool for the design and modelling activity of a typical fluid transportation pipeline. The relevance of this mathematical model is that it will guarantee secure analysis and virtual prototyping. In fact virtual prototyping is the way to go because there is always the need to ensure designs perform efficiently in the field before physical models are setup. Previously, in the absence of analysis tools, this task was only done by performing expensive and time-consuming product development cycles that involve building the model, building a physical prototype of the design, testing the prototype in the field, evaluating the results of the field tests and modifying the design based on results of the field tests. But with a mathematical model carefully crafted into a DSM tool, analysis can be done virtually to reduce cost by simulating the testing of the model on the computer instead of expensive field tests, reduce time to market by reducing the number of product development cycles, and improve products by quickly testing many concepts and scenarios before making a final decision.

2.0 Related Work

Schiozer et al. (2019), in their work 'Model-based Decision Analysis Applied to Petroleum Field Development and Management', were able to use domain specific modelling methodology to perform multi-level matching procedures by integrating geostatistical and dynamic production and pressure data. Since fluid flow dynamics were not captured, there was however the challenge of getting accurate reservoir information to provide a set of matched models for production forecasts. They applied uncertainty principle in domain specific modelling (DSM) to oil and gas reservoir management. Their objectives are to provide valuable insights for the expected value of information in a reservoir, to integrate reservoir and production systems to ensure realistic production forecasts, to adopt DSM to be able to reliably predict field performance, and to ease restrictive computational time. They basically used Domain Specific Modelling (DSM) to oil and gas reservoir management. The DSM ensures the best performance under uncertainty. It provides valuable insights for the expected value of

information and flexibility analyses. Reservoir and production systems were easily integrated to ensure realistic production forecasts. This methodology uses reservoir simulation models to reliably predict field performance. The methodology is efficient, easy-to-use and compatible with real-time operations, even in complex cases where the computational time is restrictive. The identified gap is that the dynamics of effective fluid flow into the reservoir was not captured using domain specific modelling.

Japheth and Cyril (2019) presented a Domain Specific Modeling Language Semantic Model for Artefact Orientation. The challenge was that modeling processes through domain specific language formalisms are highly volatile due to dependencies on domain concepts or used process models. But integration capabilities and necessities in a common structure and semantics were established from models explained within the oil and gas pipelines industry for artefact-orientation. The form of research basically is the application of domain semantics in DSM structure for Artefact Orientation. The objectives are: to create support for customizable development in DSM language formalism, to provide a software solution for the integration of oil and gas domain semantics that contribute a metamodel for artefact-orientation, and to provide a capable platform that stresses on the results rather than expressing a strict dependence on complicated development. They adopted the Domain Specific Modelling Languages (DSMLs) methodology for oil and gas artefact-orientation. In the findings, domain-specific method was adopted for producing artefacts without having to take into account the complexity and variability of platforms for model definitions. The identified gap is that design scenarios for the individual artifact (model) selection criteria carefully represented in a mathematical model were not explained.

A mathematical model for enhanced gas production was developed by (Yakovlev et al., 2019). In their work, the input data used in the model are the planned gas production in the reservoir and reservoir characteristics. The challenge of the application of reusable data, which was possible with layers of reusable software in DSM was identified. The form of research is the application of optimization algorithms to maximize gas recovery from a natural source. The objectives are: to create a mathematical model for 2D filtration of a two-phase multicomponent hydrocarbon mixture, to optimizing distribution of well flow rates, and to maximize gas recovery and increase gas production from natural gas reservoir. Mathematical modeling and optimization theory methods were used in this research for optimal gas production in the oil and gas industry. The findings indicated that the flow rate optimization in individual wells was carried out in such a way that the pressure drop is being kept constant for the recharge area of each well. The proposed optimization algorithm had taken into account both the restriction on

the flow rates in certain wells and their shutoffs for remedial maintenance. The identified gap is that, though mathematical modeling and optimization theory was used for optimal gas production, stakeholders design intents for possible adjustments were not captured.

Kirch et al. (2020), carried out Multiscale Molecular Modeling for enhanced oil recovery processes. The challenge was how the process could contribute to optimized petro physical processes design. The form of research was the application of molecular dynamics and finite elements modeling to increase oil recovery. The objectives are: to review recent multi-scale molecular modeling studies applied to the upstream Oil & Gas segment, to provide a suitable representation of the multiscale phenomena through molecular modeling for enhanced oil recovery and to describe the energetics of fluid and rock interacting details accurately. Finite Elements Method and Dynamics Simulations were applied in the oil and gas industry for increased oil production. The findings indicated that the outputs served as modeling descriptors at mesoscale to simulate the oil displacement by fluid injection on pore network models. The identified gap is that no clear orientation of artifacts attached to a typical oil and gas industry sector were identified, which could distort the expected flow of the oil production process.

Japheth and David (2017), presented definitions possible with a domain specific language (DSL) for Transmission Pipeline Systems Meta-Modeling. In the constructs, the specification primitives represented abstractions and conceptual modeling processes in the design and implementation of transmission pipeline configurations. The challenge was however centered on the requirements engineering framework that could seamlessly capture stakeholders competing design intents. The form of research was the application of Domain Specific Languages to oil and gas transmission pipeline metamodeling. The objectives are: to reengineering pre-constructed notations and abstractions of the pipeline engineering domain, to create a required formality that can provide expressive power in a domain specific language (DSL), and to describe specific properties that represents domain concepts, which will be useful in creating the semantic mappings of the DSL modeling platform. The Domain Specific Modelling (DSM) to oil and gas transmission pipeline design management was used. The findings are that the conceptual DSL definition brings to bear domain abstractions by taking advantage of specific properties of pipeline engineering applications that pertain to transmission. The identified gap is that wholesome design capabilities that span through both the upstream and the downstream sectors were not clearly represented.

3.0 Network Simulation (Computational Solid Mechanics)

Network simulation is a design analysis system based on 'computational solid mechanics' approach using finite element method (FEM). It is a part of the mathematical model that is fully integrated with the DSM tool in order to enhance the effective design and modelling of fluid transportation pipeline on the computer (Denis et al., 2019). The integration of the network simulation system will provide simulation solutions for linear and nonlinear static analysis, frequency analysis, buckling analysis, thermal analysis, fatigue analysis, pressure vessel analysis, linear and nonlinear transient analysis and optimization analyses. Powered by fast and accurate solvers, this system will enable pipeline engineers and stakeholder designers solve large problems intuitively while designing.

Accordingly, FEM is accepted as the standard analysis method in this context due to its generality and suitability for computer implementation, especially for mechanics of solid. It is a numerical technique used for providing approximate solutions to a set of partial differential equations that describe physical phenomena. Finite element method is a numerical technique for analyzing engineering designs, and can divide the model into many small pieces of simple shapes called elements that effectively replace a complex problem by many simple problems that need to be solved simultaneously.

In mathematical terms, finite element analysis (FEA), which is also known as the finite element method (FEM) encompasses some relevant basic steps that can improve the functionality of the DSM tool. The basic steps include geometric representation, discretization of geometry, element formulation, assembly, solution of equations, and post processing. Geometric representation creates the geometric features of the system to be analyzed and stored in a CAD database. Discretization of geometry splits the geometry into relatively small and simple geometric entities, called finite elements. This discretization process is better known as mesh formulation. The elements are called "finite" to emphasize the fact that they are not infinitesimally small, but only reasonably small in comparison to the overall model size. Element formulation develops the equations that describe the behavior of each element. Material properties for each element are considered in the formulation of the governing equations. This involves choosing a displacement function within each element. Linear and quadratic polynomials are examples of frequently used functions in this regard. Assembly obtains the set of global equations for the entire model from the equations of individual elements. The loads and support (boundary) conditions are applied to the appropriate nodes of the finite element mesh. Solution of equations provide the solutions for the unknown nodal

degrees of freedom (or generalized displacements), and post processing obtains the visualization plots for quantities of interest, such as stresses and strains, etc.

An example thermal analysis that can be available in the software system logic is as shown in the equations of heat transfer. This buttresses the need for the use of approximate methods of solution to obtain results for more complex structures. Given the equations of heat transfer: Let q_x, q_y, and q_z be the heat flux in a body in the x, y and z directions and Q the internally generated heat flow.

Then the heat balance equation is given by

$$Q - \frac{\partial q_x}{\partial x} - - \frac{\partial q_y}{\partial y} - - \frac{\partial q_z}{\partial z} = c\rho \frac{\partial T}{\partial t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

Where T = temperature

c = specific heat

ρ = mass density

The Fourier heat conduction equation relates heat flux and temperature for a thermally orthotropic body with axes of orthotropy coinciding with the coordinate axes as

$$qx' = - kx' \frac{\partial T}{dx'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

$$qy' = - ky' \frac{\partial T}{dy'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

$$qz' = -kz' \frac{\partial T}{dz'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4$$

These equations can be expressed in matrix form as

$$\begin{bmatrix} q_{x'} \\ q_{y'} \\ q_{z'} \end{bmatrix} = [\,k'\,] \begin{bmatrix} \frac{\partial T}{\partial_{x'}} \\ \frac{\partial T}{\partial_{y'}} \\ \frac{\partial T}{\partial_{z'}} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 5$$

With k' as

$$[\,k'\,] = \begin{bmatrix} k_{x'} & 0 & 0 \\ 0 & k_{y'} & 0 \\ 0 & 0 & k_{z'} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

The three components of heat conduction can be considered to be components of the heat flux vector q. Similarly, the three derivatives of T are the three components of the temperature gradient vector. Under a rotation of the coordinate system, then, the Fourier heat conduction relations become

$$\begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = - \mathrm{k} \begin{bmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{bmatrix} \quad \text{.....7}$$

Where

$$[\mathrm{k}] = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{xy} & k_{yy} & k_{yz} \\ k_{xz} & k_{xz} & k_{zz} \end{bmatrix} \quad \text{.....8}$$

With N the rotation matrix. When the coordinate axis rotation consists of a clockwise rotation through an angle θ about the z-axis from equation 8, the rotation matrix becomes

$$\mathrm{N} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{.....9}$$

And $[\mathrm{k}] =$

$$\begin{bmatrix} k_{x'}\cos^2\theta + k_{y'}\sin^2\theta & (k_{x'} - k_{y'})\sin\theta\cos\theta & 0 \\ & k_{x'}\sin^2\theta + k_{y'}\cos^2\theta & 0 \\ symetric & & k_{z'} \end{bmatrix}$$

.....10

An equivalent vibrational principle valid at every instant of time by adopting the finite element method may be written as

$\delta\pi_T = 0$ 11

Where:

$$\begin{bmatrix} \\ \\ \\ \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

$$\pi_r = \iiint (\tfrac{1}{2} \begin{bmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{bmatrix} k \begin{bmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{bmatrix} - QT - \rho cT)\, dxdydz +$$

$$\int_{sq} q^* T ds + \int_{sc} h(\tfrac{1}{2}T^2 - T T_\infty) ds \quad \ldots\ldots\ldots\ldots\ldots\ldots 12$$

And ds is an element of area of the surface S. In this formulation, $\frac{\partial T}{\partial t}$ is not subject to variation. The body is now divided into elemental sub regions and the temperature field within the element is represented by

$$T = \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} T_n \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots 13$$

Where D is a row vector of interpolation or shape functions which depend on the position in the element and $\{T_n\}$ is a column vector of nodal temperatures and possible derivatives of the temperatures. The shape function need only satisfy C° continuity, i.e. only the function itself need be continuous at element boundaries (Aziake et al., 2019).

Predicting pipeline temperature profile has become increasingly important both in the design and operation of pipelines. To predict the temperature profile and to accurately calculate pressure drop, it is necessary to divide the pipeline into smaller segments and the temperature changes can be iteratively calculated since temperature and pressure must be known at each point (segment) to compute energy balance. Also, the pipeline outer environmental soil data and temperature vary along the pipeline route and therefore play a significant role in the model in order to provide a reliable temperature profile evaluation. Figure 2 illustrates a gas flowing inside a pipe having an insulated layer.

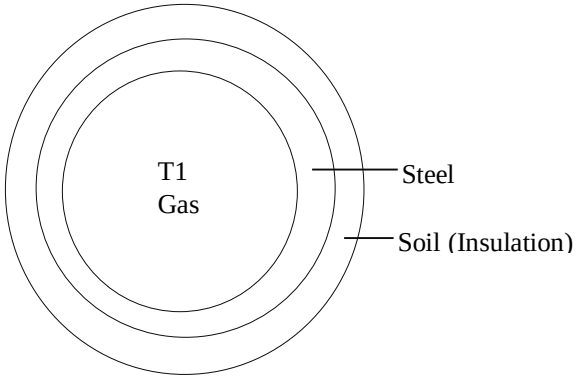

Figure 2. Gas Flowing Inside a Pipe with an Insulated Layer

Therefore, the heat transferred between the gases flowing inside a pipe having an insulated layer as shown in figure 2, is modeled to have three interfaces. Now writing the energy balance at each interface presents the following linear equations.

Heat transfer from gas to pipe:

$$U_i \pi D_1 (T_g - T_1) = \frac{T_1 - T_2}{ln(D_2/D_1)/(2\pi k_p)} \quad \ldots\ldots\ldots\ldots 14$$

Heat transfer from pipe to insulation:

$$\frac{T_1 - T_2}{ln(D_2/D_1)/(2\pi k_p)} = \frac{T_1 - T_2}{ln(D_3/D_2)/(2\pi k_I)} \quad \ldots\ldots\ldots\ldots\ldots\ldots 15$$

Heat transfer from insulation to soil:

$$\frac{T_2 - T_3}{ln(D_2/D_3)/(2\pi k \quad)} = U_o \pi D3\ (T3 - T5) \quad \ldots\ldots\ldots\ldots 16$$

Where U^i = convective heat transfer coefficient of gas (W/m².k,)

U_o = convective heat transfer coefficient of soil lagging (W/m².k)

D_1=inside pipeline diameter (mm)

D_2=outer pipe diameter (mm)

D_3=diameter of insulation (mm)

$\frac{D_3 - D_2}{2}$= Thickness of insulation

Tg = temperature of the gas (°C)

T1 = temperature of inside wall of pipe (°C) –unknown

T2 = temperature of outside wall of wall (°C) –unknown

T3 = temperature of outside of insulation (°C) –unknown

T5 = temperature of surrounding soil (°C) −unknown

Rearranging the above three energy balance equation will give the following linear algebraic equations which can be solved simultaneously to determine the values T1, T2 and T3.

$$\left[\frac{2k_p}{in(\frac{D_2}{D_1})} + U_i D_1\right] T_1 - \left[\frac{2k_p}{in(\frac{D_2}{D_1})}\right] T_2 = U_i D_1 T_g \quad \ldots 17$$

$$\left[\frac{K_p}{\ln D_2/D_1}\right] T_1 - \left[\frac{K_p}{\ln D_2/D_1} + \frac{K_i}{\ln D_3/D_2}\right] T_2 + \left[\frac{K_p}{\ln D_3/D_2}\right] T_3 \quad \ldots 18$$

$\left[\frac{2K_i}{\ln(D_3/D_2)}\right] T_2 - \left[\frac{2K_i}{\ln(D_3/D_2)} + \cup_0 D_3\right] T_3 = -\cup_0 D_3 T_5$ …...19

Kp = mean thermal conductivity of the pipe (W/m.k)

Ki = mean thermal conductivity of the insulation (W/m.k)

Temperature of the gas at a distance x from a predetermined point of the pipeline T_x was obtained by simultaneously solving the thermal, mechanical energy balances, and mass balance for the gas flow in pipelines.

$T_x = \left\{T_1 - \left[T_5 + \left(\frac{\eta}{a}\right)\left(\frac{dp}{dx}\right)\right]\right\} e^{-ax} + \left[T_5 + \left(\frac{\eta}{a}\right)\left(\frac{dp}{dx}\right)\right]$ .. 20

Where Tx = gas temperature at point x2 (°C)

T1 = initial gas temperature at point x1 (°C)

T5 = average undisturbed soil temperature at pipe centerline (°C).

η = Joule – Thompson coefficient (°C/kg/cm²)

$\eta \frac{dp}{dx}$ = Joule-Thompson effect (°C/m of pipe)

$\frac{\eta \frac{dP}{dx}}{a}$ = Temperature difference between the ground and gas which would be necessary to hold the gas temperature constant (°C).

X = $x_2 - x_1$…….21

x_2 = Distance to T2 from initial point (m)

x_1 == Distance to T1 from initial point (m)

$a = \frac{2\pi RU}{qC_p}$ ……..……..…22

Where

R = Pipe radius (cm)

U = Overall heat transfer coefficient (J/hr/°C/cm²)

q = gas flow rate (m³/hr)

Cp = specific heat of gas at constant pressure (J/°C/m³)

But $\frac{dp}{dx} = \left(\frac{dp}{dx}\right)$elev + $\left(\frac{dp}{dx}\right)$fric + $\left(\frac{dp}{dx}\right)$acc …….23

Where $\left(\frac{dp}{dx}\right)$ = flow pressure gradient

$\left(\frac{dp}{dx}\right)$elev = $\frac{\rho g}{g_c} \sin\theta$ ……..……24

$\left(\frac{dp}{dx}\right)$fri= $\frac{\rho f v^2}{2g_c \Delta}$ ……25

$\left(\frac{dp}{dx}\right)$acc = $\frac{\rho f}{gc}\left(\frac{dv}{dx}\right)$ ……..……26

The pressure drop component caused by acceleration is normally negligible and is considered only for cases of high velocity.

Where x = flow length (m)

ρ = gas density (kg⁄m3)

V = flow velocity (m⁄s2)

f = frictional coefficient

D= internal pipeline diameter (mm)

θ = inclination angle of pipeline (°)

g = gravitational acceleration (m⁄s²)

gc = gravitational constant

4.0 Network Flow Simulation (Computational Fluid Mechanics)

The flow simulation component is another sub-program to be fully integrated with the other components of the DSM tool. It is basically for computing fluid flows by solving the time-dependent Navier-Stokes equations through iterations in the network models, as well as heat transfers to and from these models due to convection, radiation, and conduction with a proved computational fluid dynamics (CFD) technology (Ariza, and Jiménez-Cabas, 2018).

Flow simulation, being a part of the DSM tool mathematical model will allow intended users to analyze a wide range of complex flow problems involving flows having two-and three-dimensional characteristics, external and internal (in and/or over solids), steady-state and transient properties, laminar, turbulent, and transitional routing in rotating devices e.g. pumps etc. Also involving fluids with multi-species characteristic, gases, and incompressible and compressible liquids. The flow simulation phenomena include gravitational (buoyancy) effects, gas relative humidity, condensate flows, condensation in vapor flows, cavitation in liquid flows, tracers, liquid or solid particles in a carrying fluid, and heat transfer within fluids, between fluids and solids.

As mentioned earlier, flow simulations solve the Navier-Stokes equations, which are conservative representations of mass, momentum and energy laws for fluid flows. Both turbulent and laminar flows will be adequately predicted by network flow simulator (Aziaka, 2020). The conservation laws of mass, momentum and energy in a Cartesian coordinate system rotating with an angular velocity Ω about an axis passing through the coordinate system's origin can be written in the conservative form as:

$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(eu_i) = S_M^p$ ……..……27

$\frac{\partial \rho U_i}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho U_i U_j\right) + \frac{\partial p}{\partial x_i} = \frac{\partial}{\partial x_j}\left(t_{ij} + t_{ij}^R\right) + S_i + S_{ij}^P \; i = 1,2,3$

…… 28

$\frac{\partial \rho H}{\partial t} + \frac{\partial \rho U_i H}{\partial x_i} = \frac{\partial}{\partial x_i}\left[U_i\left(t_{ij} + t_{ij}^R\right) + q_i\right] + \frac{\partial p}{\partial t} - t_{ij}^R \frac{\partial U_i}{\partial x_j} + \rho \in + S_i U_i + S_H^P + Q_H$

…….. 29

Where $H = h + \frac{U^2}{2} + \frac{5}{3}k - \frac{\Omega^2 r^2}{2} \sum_m h_m^{\circ} y_m$ ……30

u = fluid velocity

ρ = fluid density

S_i= mass distributed external force per unit mass due to a porous media resistance(S_i^{Porous}), buoyancy $\left(S_i^{gravity}\right) = eg_i$ and rotation $\left(S_i^{rotation}\right)$

$S_i = S_i^{porous} + S_i^{gravity} + S_i^{rotation}$ ……..……31

h = thermal enthalpy

$S_{M,}^P \; S_{Ii}^P, S_H^P$ = additional interfacial exchange terms due to Euler- Lagrange particle interaction.

Q_H = heat source per unit volume

t_{ij} = viscous shear stress tensor

q_i = diffusive heat flux

Ω = angular velocity of the rotating coordinate system

r = distance from a point to the rotation axis in rotation frame

k = kinetic energy of turbulence

h_m° = individual thermal enthalpy of m-th mixture component

y_m = concentration of the m-th mixture component

For Newtonian fluids, the viscous shear stress tensor is defined as:

$t_{ij} = u\left[\frac{\partial U_i}{\partial x_j} + \frac{\partial U_i}{\partial x_i} - \frac{2}{3}\sigma_{ij}\frac{\partial U_k}{\partial x_k}\right]$ ……32

And following Boussineq assumption the Reynolds – stress tensor can be represented as:

$t_{ij}^R = \rho u\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} - \frac{2}{3}\sigma_{ij}\frac{\partial U_k}{\partial x_k}\right) - \frac{2}{3}\rho k\,\sigma_{ij}$-…… 33

Where σ_{ij} = Kronecker delta function

μ = dynamic viscosity coefficient

μ_t = turbulent eddy viscosity coefficient

K = turbulent kinetic energy

ϵ = turbulent dissination

But $\mu_t = f_u \frac{C_u \rho k^2}{\epsilon}$ ……..…34

Where f_μ = turbulent viscosity factor

And $f_\mu = \left[1 - exp\left(-0.0165 R_y\right)\right] \cdot \left(1 + \frac{20.5}{R_T}\right)$ …….. 35

Where $R_T = \frac{\rho k^2}{\mu \epsilon}$..…….....… 36

$R_y = \frac{\rho \sqrt{ky}}{\mu}$ ……..... 37

y = Distance from the wall

5.0 Modelling the Transient Flow in Liquid Pipeline

The software can proffer solution for both static and transient analysis. This section gives an example of how transient analysis can be done with the software. This can be achieved through discretization process, where each term represented in the combined equations is translated into a numerical analogue that can be programmed to solve (. Japheth et al., 2020). The key features of the mathematical model in the DSM tool in this regard shall include free-form 3d modelling, a complete set of 2d drafting, good annotation tools compatibility with other design, drafting, rendering and animation software, and a built-in part library.

Changes in the system flow in a fluid in a pipe often course sudden pressure changes and give rise to so called transient load flows. When transient flow occurs in a pipe network, it will result in negative pressures thus damaging the pipeline. The flow in the pipeline can be regarded as one dimensional as fluid movement must satisfy mass, momentum and energy conservations (Nazar at al., 2005). Thus, the model of fluid movement in pipe can be based on the dynamics of the continuity, momentum and energy equation which adequately describes the relationship between pressure, temperature and flow in the pipe network. These are re-written as:

$\frac{\partial P}{\partial t} + \frac{V \partial P}{\partial x} + \rho a^2 \frac{\partial V}{\partial x} = 0$ ……..…. 38

$\frac{\partial V}{\partial t} + \frac{1}{\rho} \frac{\partial P}{\partial x} + V \frac{\partial V}{\partial x} = -g \sin\theta - \frac{\lambda}{2D} V|V|$ ……..... 39

$\frac{\partial T}{\partial T} + \frac{T}{\rho C} \left(\frac{\partial P}{\partial T}\right)_\rho \frac{\partial V}{\partial x} + V \frac{\partial T}{\partial x} = \lambda \frac{|V|^3}{2DC} - \frac{4K}{\rho DC} (T - T_\circ)$……..40

Where ρ = fluid density

P = pipe pressure

θ = inclination of the pipeline

t = time variable

V = fluid (velocity)

D = internal diameter of the pipe

C = heat capacity

g = acceleration due to gravity

x = pipe position variable

K = total heat transfer coefficient

T = medium temperature

$T_{\circ}$ = internal (ground) temperature

λ = pipe line hydraulic function coefficient

For higher precision and better stability, the explicit characteristics differences methodology is adopted. This implies that the characteristics value 7λ from linear algebra

$$A = \begin{bmatrix} V & \rho a^2 & 0 \\ \frac{1}{\rho} & V & 0 \\ 0 & \frac{T}{\rho C}\left(\frac{\partial P}{\partial T}\right)_{\rho} & V \end{bmatrix} \text{……..…. 41}$$

Must satisfy

$|\lambda I - A| = 0$ ……...…42

Where $I =$ third order unit matrix and $|\lambda I - A| =$ third order determinant.

Thus, the Eigen values from the above equation gives

$$\left.\begin{aligned} \lambda_1 &= V + a \\ \lambda_2 &= V - a \\ \lambda_3 &= V \end{aligned}\right\} \text{……..…. 43}$$

And the corresponding Eigen vectors for each Eigen value λ_i is

$$\left.\begin{aligned} L^{(1)} &= (1, \rho a, 0) \\ L^{(2)} &= (1, -\rho a, 0) \\ L^{(3)} &= \left(-\frac{T}{\rho C}\left(\frac{\partial P}{\partial T}\right)_{\rho}, 0, 1\right) \end{aligned}\right\} \text{..... 44}$$

Hence equations (38), (39), and (40) can be estimated as follows:

First the positive characteristics line equation C+

$$\left.\begin{aligned} \frac{dx}{dt} &= V + a \\ \frac{dV}{dt} + \frac{1}{\rho a}\frac{dP}{dt} &= -g\sin\theta - \frac{\lambda}{2D}V|V| \end{aligned}\right\} \text{……..…… 45}$$

Secondly, the negative characteristics line equation C- yields

$\frac{dx}{dt} = V - a$---……..…… 46a

$$\frac{dV}{dt} - \frac{1}{\rho a}\frac{dP}{dt} = -g\sin\theta - \frac{\lambda}{2D}V|V| \text{……...….46b}$$

Thirdly, the temperature characteristic linear equation V gives

$$\left.\begin{aligned} \frac{dx}{dt} &= V \\ \frac{d(CT)}{dt} - \frac{T}{\rho}\left(\frac{\partial P}{\partial T}\right)_{\rho}\frac{\partial V}{\partial x} &= +\lambda\frac{|V|^3}{2D} - \frac{4k}{\rho D}(T - T_{\circ}) \end{aligned}\right\} \text{- ……47}$$

The temperature characteristics equation can be re-written as follows by substituting the continuity equations into equation (47).

$$\left.\begin{aligned} \frac{dx}{dt} &= V \\ \frac{d(CT)}{dt} - \frac{P}{\rho^2 a^2}\frac{dP}{dt} &= +\lambda\frac{|V|^3}{2D} - \frac{4k}{\rho D}(T - T_{\circ}) \end{aligned}\right\} \text{- ……. 48}$$

The solution above the characteristic linear mathematical model of the DSM tool using the three different characteristic directions ($\lambda_1 = V, \lambda_2 = V + a \; and \; \lambda_3 = V - a$), transforms the partial differential equations into ordinary differential equations.

Now considering a pipeline length of L, λ as data collection cycle and the pipeline divided into n parts with pipe step length Δx and time step length Δt to disperse $x - t$ plane grid, is indicated in figure 3.

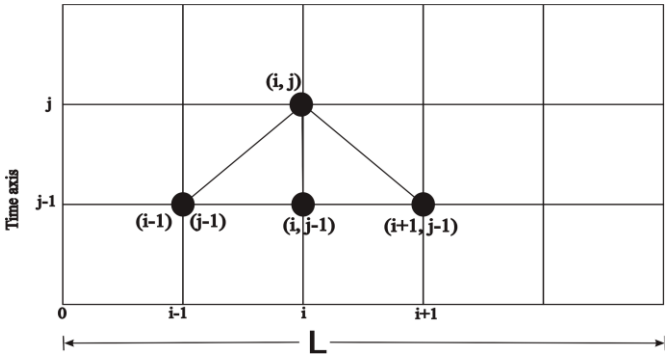

Figure 3 Pipeline Data Collection Cycle

This represents the simulation for the boundary temperature, pressure and flow. Using a central difference, the central difference quotient equation of the characteristic equations can be as in equations (45), (46) and (47):

C+: $\frac{V_{i,j} - V_{i-1,j-1}}{\Delta t} - \frac{P_{i,j} - P_{i-1,j-1}}{\rho a \Delta t} + g\sin\theta + \frac{\lambda}{8D}\left(V_{i,j} + V_{i-1,j-1}\right)^2 = 0$

…….. 49

C-: $\frac{V_{i+1,j-1} - V_{i,j}}{\Delta t} - \frac{P_{i+1,j-1} - P_{i,j}}{\rho a \Delta t} + g\sin\theta + \frac{\lambda}{8D}\left(V_{i+1,j-1} + V_{i,j}\right)^2 = 0$

…….... 50

V: $C\frac{T_{i,j} - T_L}{\Delta t} - \frac{P_{i,j} + P_L}{2\rho^2 a^2}\frac{P_{i,j} - P_L}{\Delta t} = \lambda\frac{\left(V_{i,j} + V_L\right)^3}{16D} - \frac{2k}{\rho D}\left(T_{i,j} + T_L - 2T_{\circ}\right)$-

……..……51

Where

$$V_L = \frac{V_{i,j-1}}{1-(V_{i-1,j-1}-V_{i,j-1})\frac{\Delta t}{\Delta x}} \text{ --}\ldots\ldots\ldots\ldots\ldots\ldots\ldots 52$$

$$P_L = \left(P_{i-1,j-1} - P_{i,j-1}\right)\frac{\Delta t}{\Delta x}V_2 + P_{i,j-1}\ldots\ldots\ldots\ldots\ldots\ldots 53$$

$$T_L = \left(T_{i-1,j-1} - T_{i,j-1}\right)\frac{\Delta t}{\Delta x}V_L + T_{i,j-1}\ldots\ldots\ldots\ldots\ldots 54$$

These outcomes combined with boundary conditions yields the next time layer values for $V_{i,j}, P_{i,j}$ and $T_{i,j}$ as follows:

$$V_{i,j} = \frac{V_{i-1,j-1} + V_{i+1,j-1} - \frac{P_{i+1,j-1}-P_{i-1,j-1}}{\rho a} - \frac{\lambda\Delta t}{8D}\left(V_{i-1,j-1}^2 - V_{i+1,j-1}^2\right)}{2+\frac{\lambda\Delta t}{4D}(V_{i-1,j-1}-V_{i+1,j-1})}$$

…….. 55

$$P_{i,j} = \frac{V_{i-1,j-1}+V_{i+1,j-1}}{2}\rho\text{a} + \frac{P_{i+1,j-1}+P_{i-1,j-1}}{2} -$$

$$-\frac{\lambda\rho a\Delta t}{16D}\left[\left(V_{i-1,j-1}+V_{i,j}\right)^2\left(V_{i+1,j-1}+V_{i,j}\right)^2\right]\rho g\Delta x\sin\theta$$

…….56

$$T_{i,j} = \frac{\frac{P_{i,j}+P_L}{2\rho^2 a^2}\frac{P_{i,j}-P_L}{\Delta t} + \frac{\lambda\left(V_{i,j}+V_L\right)^3}{16D} - \frac{2k(T_L-2T_\circ)}{\rho D} + \frac{T_L C}{\Delta t}}{\frac{C}{\Delta t}+\frac{2k}{\rho D}} \ldots\ldots 57$$

The mathematical model presented through the Navier-Stroke equation used is seen to accurately and amazingly models a whole set of flow phenomena from turbulent or laminar, single phase incompressible flows, to compressible all-speed, and all the way to multiphase flows (Pavlenko et al., 2020).

6.0 Conclusion and Future Work

The software capabilities span through the transient effect of temperature, pressure, stress, corrosion allowance for pipe, buckling effects for a successful flow assurance simulation and operation of a pipeline. This can be achieved through a mathematical model encompassing pipeline network design and modelling of transient flow in fluid pipeline, pipeline network simulation using computational solid mechanics, and pipeline network flow simulation using computational fluid mechanics. The key processes of the software implementation will start from problem definition to visualization of simulated results. Basically it starts from formulating the flow problem, modelling the geometry and flow domain, establishing the boundary and initial conditions, generating the grid/mesh, establishing the simulation strategy, establishing the input parameters and files, performing the simulation, monitoring the

simulation for completion, post-processing of the simulation to get the results, making comparisons of the results, and repeating the process to examine sensitivities, and documentation. The mathematical model when fully constrained with the domain concepts, parameters, attributes and values will clearly predicted accurate modelling of a whole set of flow phenomena from turbulent or laminar, single phase incompressible flows, to compressible all-speed, and all the way to multiphase flow. Therefore the simulation results and pseudocodes representing the type semantics for each scenario shall be showcased in in future work.

In the future instead of the Navier-Stroke equation used, a more accurate solution methodology will be the finite volumes method (FVM). In FVM, some of the terms in the conservative equations are turned into face fluxes and as thus evaluated at the finite volume faces. This inherent conservation property of the FVM makes it the preferred method in computational fluid dynamics (CFD). Another attribute of the FVM is that it can be formulated in the physical space on unstructured polygonal meshes. Finally, it is quite easy to implement a variety of boundary conditions in a non-invasive manner, since the unknown variables are evaluated at the centroids of the volume element and not at their boundary faces. Thus, this method will be greatly utilized in further enhancing the capacity of the DSM tool in computation network flow simulation.

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