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Load Prediction on Metal Forming Process (Forging) Using Finite Element Method.

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ABSTRACT

Metal forming process is a widely used manufacturing process especially in high volume metal production system. In this paper, the main objective is using Bubnov-Galerkin finite element model to derive the pressure field set up at various cross-sections of a metal blank during a forging process, and the four Lagrange quadratic elements were assembled to represent the various metal blank. The governing equation adopted for this paper is a one-dimensional differential equation describing the pressures exerted on the forging process. During the analysis, the various metal blanks are divided into a finite number of elements and the weighted integral form for each element were formed after applying the Bubnov-Galerkin weighted residual method. A matrix form under certain boundary conditions from the weighted residual method were used to obtain the pressure distribution across the cross-section of the various metal blanks. Finite element results are obtained for a value of a circular disc diameter, thickness, coefficient of friction, principal stress, length, and radius of a circular material. Finite element method and the Exact solution approach are used to achieve and compare both results. Furthermore, the combination of both methods shows that there are potentials for using this approach towards the optimization of metal forming in manufacturing processes and some engineering practices.

Keywords: Forging; LaGrange Interpolation Function; Bubnov-Galerkin Weighted Residual Method; Finite Element Method.

1. INTRODUCTION

Over the decades, metal forming process has been a widely used manufacturing process especially for a high-volume metal production system. The main goals of engineering analysis are to be able to identify the basic physical principle that governs the behaviour of systems, also to translate these principles into several mathematical models that comprise of most differential equations that can be solved to accurately predict various forms of system behaviour (Zhang and Wang, 2016). These behaviours include but are not limited to metal forming stability, rapid change in pressure, and instability in design of other metal forming processes. Similarly, different methods of solving mathematical equations exist with the aid of model simulation, which includes using either finite element method (Song and Im, 2007; Bickford, 1990; and Burnet, 1987). The most unique features of the finite element method that separates it from other methods, is the division of a given domain into an asset of simple sub-domains called finite elements. According to Reddy (1984), any geometrical shape that allows computation and approximation of its solution or provides necessary relationship among the values of the solution at a selected point known as nodes of the sub-domains is called finite element.

In other words, finite element method (FEM) is one of the most distinctive methods to simulate any metal forming processes. Also, FEM is a numerical method used in solving problems by the decentralization of a given domain into sub-domains. In addition, FEM has been used by many users in manufacturing for the optimization of die design process, in order to derive design parameters without damaging any physical structure. This physical structure can easily be modeled using a various computer-aided design (CAD) package. According to Castro et al., (2004), optimal design in forging using FEM was developed based on an evolutionary strategy (i.e., genetic algorithms). Furthermore, a rigid viscoplastic flow-type formulation was also adopted, which is valid for both hot and cold forging process. In industrial forming processes most of the deformation energy is transformed into thermal energy, thereby generating heat which causes the increase in temperature and external friction losses within the die–work-piece interface to obtain optimal solutions.

According to Kim et al. (1996), several reviews on cold-forged metal parts were collected both from the industries and secondary sources. Based on their report, it was accounted that to achieve a verified metal forming process sequence, the finite element simulation program such as the use

of DEFORM (MacCormack and Monaghan, 2001) can be used to simulate the behaviour of any specific forged parts. Wang et al. (2007) developed three design schemes with different die shapes. Firstly, FEM is used to simulate the cold forging process of the spur gear with a two-dimensional axisymmetric model, and the strain distributions and velocity distributions are investigated through the post-processor. Radial-flow-velocity distribution is an important indicator to be evaluated, and a relatively better scheme is selected. Secondly, three-dimensional simulation for the relative scheme is further performed considering the complicated geometric nature of gear, and the results show that the corner filling is improved, and a well-shaped gear is forged. Finally, a corresponding experiment is done, which is mainly utilized for supporting and validating the numerical simulation and theoretical investigation. Kim et al., (2003) have used rigid-plastic finite element simulation to analyse the deformation characteristic of the whole impeller hub forming processes and to optimize the process. As a result, two kinds of improvement for the impeller hub forming process satisfying the limit of the machine's load capacity and the geometrical quality are suggested and they verified their findings with experimental results.

Akpobi and Edobor (2009) used finite element analysis with Lagrange interpolation function to analyse the distribution of velocity in viscous incompressible fluids. Their results show that as the number of elements increases, so is the accuracy of the finite element solution compared with the exact solution. Erhunmwum and Oladeinde (2016) use finite element method as a computational modelling to analyse the dynamic behaviour of the velocity distribution of an incompressible fluid flowing through a cylindrical annulus pipe. In their results, FEM result were compared with the exact solution.

This paper will present a numerical analysis of a metal forming process (i.e., a closed die forging) by using FEM. To realize the goal of this research, specific objective was proposed, which is to numerically investigate the load prediction on a plain strain forging workpiece by considering the external force acting on each element and the width dx using finite element method. To achieve this, Bubnov-Galerkin weighted-residual method was used. The numerical solution of the method adopted were compared with the analytical solution.

In the remaining part of the paper, Section 2 will present the numerical method to justify the need of the proposed research objective. In Section 3, the result analysis on forging process were

presented. In Section 4, results were discussed. Finally, Section 5 presents the conclusions and suggestions for future research.

2.0: MATERIALS AND METHODS

2.1: Numerical Method

In order to describe the behaviour of a forging process, the external force acting on each finite element in a forging workpiece and the width is considered. The Bubnov-Galerkin weighted-residual method was developed as a finite element mathematical modeling to predict the loads on forging work piece. Therefore, for a closed die forging at any instant, the equilibrium equation of a small element of the width dx in x direction will be,

$$(\sigma_x + d\sigma_x)Bh - \sigma_x Bh - 2\tau_x dx B = 0 \quad (1)$$

From here, we get

$$\sigma_x Bh + Bh d\sigma_x - \sigma_x Bh - 2\tau_x dx = 0 \quad (2)$$

$$Bh d\sigma_x - 2\tau_x B dx = 0$$

$$\frac{d\sigma_x}{dx} - \frac{2\tau_x}{h} = 0 \quad (3)$$

Generally, we consider a forging process where interfacial friction is involved and as such, we assume Coulomb friction with constant coefficient of friction, also we apply Tresca's yield criterion. We also set the principal stresses as;

$$\sigma_1 = \sigma_x$$

$$\sigma_3 = -p$$

$$\tau_x = \mu p_x$$

$$\text{where, } \frac{d\sigma_x}{dx} - \frac{2\mu p}{h} = 0 \quad (4)$$

σ_x = Principal Stress

$\tau_x = \mu p$ = Friction force

p = Die pressure

h = height = dx (width)

Since equation (4) is the governing equation for the forging process, we then apply the weighted integral formulation.

2.2 Weighted Integral Formulation

The weighted integral form of the governing equation in equation (4) is obtained by multiplying through by the weight function, W , and integrating over the domain enclosing an element with respect to x .

$$\int \left(W \frac{dp}{dx} + \frac{2\mu}{h} WP \right) dx \quad (5)$$

An examination of equation (4) shows that the solution and hence the approximation function should be once differentiable with respect to x .

Hence the Lagrange interpolation functions can be used satisfactorily.

Let us assume that the solution p is approximated as follows:

$$P = P^e = \sum_{j=1}^n P_j^e \psi_j^e(x) \quad (6)$$

Applying the Bubnov-Galerkin weighted-residual method to get the finite element model.

$$\text{Let, } W = \psi_j^e \quad (7)$$

$$\psi_1^e = \left(1 - \frac{4x}{L}\right) \left(1 - \frac{8x}{L}\right) \quad (8)$$

$$\psi_2^e = \frac{16x}{L \left(1 - \frac{4x}{L}\right)} \quad (9)$$

$$\psi_3^e = -\frac{4x}{L \left(1 - \frac{8x}{L}\right)} \quad (10)$$

$$\frac{d\psi_1^e}{dx} = \frac{4(16x-3L)}{L^2} \quad (11)$$

$$\frac{d\psi_2^e}{dx} = \frac{-16(8x-L)}{L^2} \quad (12)$$

$$\frac{d\psi_3^e}{dx} = \frac{4(16x-L)}{L^2} \quad (13)$$

Substituting equation (6) and (7) into equation (5)

$$0 = \int \left(\psi_i^e \frac{d}{dx} \sum_{j=1}^n P_j^e \psi_j^e + \frac{2\mu}{h} \psi_i^e \sum_{j=1}^n P_j^e \psi_j^e \right) dx \quad (14)$$

After recasting the equation (14) to form the below equation,

$$0 = \sum_{i=1}^n (K_{ij}^e) \{P_j^e\} \quad (15)$$

$$K_{ij}^e = \int_{x_A}^{x_B} \left(\psi_i^e \frac{d\psi_j^e}{dx} + \frac{2\mu}{h} \psi_i^e \psi_j^e \right) dx \quad (16)$$

The above equation (16) is the weighted residual finite element model of a forging process

$$\begin{aligned} K_{11} &= \int_0^{L/4} \left[\psi_1 \frac{d\psi_1}{dx} + \frac{2\mu}{h} \psi_1 \psi_1 \right] dx \\ K_{11} &= \int_0^{L/4} \left[\left(1 - \frac{12x}{L} + \frac{32x^2}{L^2} \right) \left(-\frac{12}{L} + \frac{64x}{L^2} \right) + \frac{2\mu}{h} \left(1 - \frac{12x}{L} + \frac{32x^2}{L^2} \right) \left(1 - \frac{12x}{L} + \frac{32x^2}{L^2} \right) \right] dx \\ &= \int_0^{L/4} \left[\left(-\frac{12}{L} + \frac{208x}{L^2} - \frac{1152x^2}{L^3} + \frac{2048x^3}{L^4} \right) + \frac{2\mu}{h} \left(1 - \frac{24x}{L} + \frac{208x^2}{L^2} - \frac{768x^3}{L^3} + \frac{1024x^4}{L^4} \right) \right] dx \\ K_{11} &= \frac{-30h + 4\mu L}{60h} \end{aligned} \quad (17)$$

$$\begin{aligned} K_{12} &= \int_0^{L/4} \left[\psi_1 \frac{d\psi_2}{dx} + \frac{2\mu}{h} \psi_1 \psi_2 \right] dx \\ K_{12} &= \int_0^{L/4} \left[\left(1 - \frac{12x}{L} + \frac{32x^2}{L^2} \right) \left(\frac{16}{L} - \frac{128x}{L^2} \right) + \frac{2\mu}{h} \left(1 - \frac{12x}{L} + \frac{32x^2}{L^2} \right) \left(\frac{16x}{L} - \frac{64x^2}{L^2} \right) \right] dx \\ &= \int_0^{L/4} \left[\left(\frac{16}{L} - \frac{320x}{L^2} + \frac{2048x^2}{L^3} - \frac{4096x^3}{L^4} \right) + \frac{2\mu}{h} \left(\frac{16x}{L} - \frac{256x^2}{L^2} + \frac{1280x^3}{L^3} - \frac{2048x^4}{L^4} \right) \right] dx \\ K_{12} &= \frac{40h + 2\mu L}{60h} \end{aligned} \quad (18)$$

$$\begin{aligned} K_{13} &= \int_0^{L/4} \left[\psi_1 \frac{d\psi_3}{dx} + \frac{2\mu}{h} \psi_1 \psi_3 \right] dx \\ K_{13} &= \int_0^{L/4} \left[\left(1 - \frac{12x}{L} + \frac{32x^2}{L^2} \right) \left(-\frac{4}{L} + \frac{64x}{L^2} \right) + \frac{2\mu}{h} \left(1 - \frac{12x}{L} + \frac{32x^2}{L^2} \right) \left(-\frac{4x}{L} + \frac{32x^2}{L^2} \right) \right] dx \\ &= \int_0^{L/4} \left[\left(-\frac{4}{L} + \frac{112x}{L^2} - \frac{896x^2}{L^3} + \frac{2048x^3}{L^4} \right) + \frac{2\mu}{h} \left(-\frac{4x}{L} + \frac{80x^2}{L^2} - \frac{512x^3}{L^3} + \frac{1024x^4}{L^4} \right) \right] dx \\ K_{13} &= \frac{-10h - \mu L}{60h} \end{aligned} \quad (19)$$

Using the above procedure, we evaluate for the other element of K_{ij}^e . The equations were then assembled for the four quadratic element meshes. The boundary condition is

$$\text{At } x = L, \sigma_x = 0$$

$$\text{But } \sigma_x + p_x = \sigma_0 = 2k$$

$$\text{Therefore, } x = L, P_9 = \sigma_0 = 2k$$

$K^e =$

$$\frac{1}{60h} \begin{bmatrix} -30h + 4\mu L & 40h + 2\mu L & -10h - \mu L & 0 & 0 & 0 & 0 & 0 & 0 \\ -40h + 2\mu L & 16\mu L & 40h + 2\mu L & 0 & 0 & 0 & 0 & 0 & 0 \\ 10h - \mu L & -40h + 2\mu L & 8\mu L & 40h + 2\mu L & -10h - \mu L & 0 & 0 & 0 & 0 \\ 0 & 0 & -40h + 2\mu L & 16\mu L & 40h + 2\mu L & 0 & 0 & 0 & 0 \\ 0 & 0 & 10h - \mu L & -40h + 2\mu L & 8\mu L & 40h + 2\mu L & -10h - \mu L & 0 & 0 \\ 0 & 0 & 0 & 0 & -40h + 2\mu L & 16\mu L & 40h + 2\mu L & 0 & 0 \\ 0 & 0 & 0 & 0 & 10h - \mu L & -40h + 2\mu L & 8\mu L & 40h + 2\mu L & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -40h + 2\mu L & 16\mu L \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} \frac{\sigma}{60h} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 10h + \mu L \\ 40h + 2\mu L \end{bmatrix} \quad (20)$$

The stresses are obtained by substituting the values of the pressure into the equation;

$$\sigma_x = \sigma_0 - p$$

3.0: RESULTS AND DISCUSION

3.1.1: Result Analysis on Forging process

To interpret the model formulation and its accuracy, two illustrative numerical examples will be used.

Example 1, for plane strain forging of a circular disc of diameter 48mm, thickness of 66mm, coefficient of friction ($\mu = 0.2$). Assume that no sticking occurs, pressure (p), with respect to the principal stress (σ_0), at the nodes are solved by the weighted residual FEM and Exact solution method.

3.1.2: Forging (Exact Solution)

To evaluate the exact forging stress in example 1 as given above, equation (21) was used to calculate the exact solution of the forging process.

$$\frac{P}{\sigma_0} = e^{(2\mu(L-x)/h)} \quad (21)$$

Let, $L = 24\text{mm}$ (radius of circular material)

Since x represents the radius of each element. It therefore assumes values of 3mm, 6mm, 9mm, 12mm, 15mm, 18mm, 21mm, 24mm, for the eight elements. Also, for each x of a given module at a height 36mm with $\mu = 0.2$, the following result was obtained for the various radius of each element as shown in Table 1.

Table 1: Comparison of radial distance and exact equation

Radius L (mm)	X (mm)	Height h (mm)	Exact Solution $\frac{P}{\sigma_0}$
24.00	3.00	36.00	1.263
24.00	6.00	36.00	1.220
24.00	9.00	36.00	1.182
24.00	12.00	36.00	1.143
24.00	15.00	36.00	1.105
24.00	18.00	36.00	1.069
24.00	21.00	36.00	1.034
24.00	24.00	36.00	1.000

Example 2: Considering the forging of a circular disc of diameter 54.97mm and thickness of 36mm with coefficient of friction $\mu = 0.2$

Table 2: Comparison of radial distance and exact equation

Radius L (mm)	X (mm)	Height h (mm)	Exact Solution $\frac{P}{\sigma_0}$
27.485	3.440	36.00	1.159
27.485	6.870	36.00	1.135
27.485	10.310	36.00	1.111
27.485	13.740	36.00	1.088
27.485	17.180	36.00	1.065
27.485	20.610	36.00	1.043
27.485	24.050	36.00	1.021
27.485	27.485	36.00	1.000

3.1.3: Finite Element Method (Forging)

Using the finite element method, a stiffness matrix and source vector was developed as shown in the equation below to obtained the pressure with respect to the principal stress distribution over each element is obtained by substituting $R = 24\text{mm}$, $h = 36\text{mm}$, $\mu = 0.2$, into the below 8×8 assembled matrix equation for example 1.

$$\frac{1}{60h} \begin{bmatrix} -30h + 4\mu L & 40h + 2\mu L & -10h - \mu L & 0 & 0 & 0 & 0 & 0 \\ -40h + 2\mu L & 16\mu L & 40h + 2\mu L & 0 & 0 & 0 & 0 & 0 \\ 10h - \mu L & -40h + 2\mu L & 8\mu L & 40h + 2\mu L & -10h - \mu L & 0 & 0 & 0 \\ 0 & 0 & -40h + 2\mu L & 16\mu L & 40h + 2\mu L & 0 & 0 & 0 \\ 0 & 0 & 10h - \mu L & -40h + 2\mu L & 8\mu L & 40h + 2\mu L & -10h - \mu L & 0 \\ 0 & 0 & 0 & 0 & -40h + 2\mu L & 16\mu L & 40h + 2\mu L & 0 \\ 0 & 0 & 0 & 0 & 10h - \mu L & -40h + 2\mu L & 8\mu L & 40h + 2\mu L \\ 0 & 0 & 0 & 0 & 0 & 0 & -40h + 2\mu L & 16\mu L \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} = \frac{\sigma}{60h} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 10h + \mu L \\ 40h + 2\mu L \end{bmatrix}$$

$$\begin{bmatrix} -0.4911 & 0.6711 & -0.1689 & 0 & 0 & 0 & 0 & 0 \\ -0.6622 & 0.0356 & 0.6711 & 0 & 0 & 0 & 0 & 0 \\ 0.1644 & -0.6622 & 0.0178 & 0.6711 & -0.1689 & 0 & 0 & 0 \\ 0 & 0 & -0.6622 & 0.0356 & 0.6711 & 0 & 0 & 0 \\ 0 & 0 & 0.1644 & -0.6622 & 0.0178 & 0.6711 & -0.1689 & 0 \\ 0 & 0 & 0 & 0 & -0.6622 & 0.0356 & 0.6711 & 0 \\ 0 & 0 & 0 & 0 & 0.1644 & -0.6622 & 0.0178 & 0.6578 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.6622 & 0.0356 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.1689 \\ -0.6711 \end{bmatrix} P = \begin{bmatrix} 1.3072 \\ 1.2643 \\ 1.2228 \\ 1.1829 \\ 1.1440 \\ 1.1066 \\ 1.0701 \\ 1.0560 \end{bmatrix}$$

Example 2: Considering the forging of a circular disc of diameter 54.97mm and thickness of 65.09mm with coefficient of friction $\mu = 0.2$

$$\begin{bmatrix} -0.4944 & 0.6695 & -0.1680 & 0 & 0 & 0 & 0 & 0 \\ -0.6640 & 0.0225 & 0.6695 & 0 & 0 & 0 & 0 & 0 \\ 0.1653 & -0.6640 & 0.0113 & 0.6695 & -0.1680 & 0 & 0 & 0 \\ 0 & 0 & -0.6640 & 0.0225 & 0.6695 & 0 & 0 & 0 \\ 0 & 0 & 0.1653 & -0.6640 & 0.0113 & 0.6695 & -0.1680 & 0 \\ 0 & 0 & 0 & 0 & -0.6640 & 0.0225 & 0.6695 & 0 \\ 0 & 0 & 0 & 0 & 0.1653 & -0.6640 & 0.0113 & 0.6695 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.6640 & 0.0225 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.168 \\ -0.669 \end{bmatrix} P = \begin{bmatrix} 1.1828 \\ 1.1581 \\ 1.1342 \\ 1.1103 \\ 1.0876 \\ 1.0645 \\ 1.0429 \\ 1.0207 \end{bmatrix}$$

The results from each example based on finite element method solution are presented in Table 3 and Table 4 in comparison with the exact method solution.

Table 3: Comparison of exact solution and finite element solution

Distance (mm)	Exact Solution		Finite Solution Pressure	% Difference
	$\frac{P}{\sigma_0}$			
3.00	1.263		1.3072	3.3813
6.00	1.220		1.2643	3.5039
9.00	1.182		1.2228	3.3336
12.00	1.143		1.1829	3.3731
15.00	1.105		1.1440	3.4091
18.00	1.069		1.1066	3.3998
21.00	1.034		1.0701	3.3735
24.00	1.000		1.0560	5.3030

Table 4: Comparison of exact solution and finite element solution

Distance (mm)	Exact Solution	Finite Solution Pressure	% Difference
	$\frac{P}{\sigma_0}$		
3.440	1.159	1.1828	2.0708
6.870	1.135	1.1581	1.9862
10.310	1.111	1.1342	2.0282
13.740	1.088	1.1103	1.9820
17.180	1.065	1.0876	2.1140
20.610	1.043	1.0645	2.0657
24.050	1.021	1.0429	2.1093
27.485	1.000	1.0207	2.0568

The values of pressures using the exact equation and the finite element methods are already tabulated. Therefore, the graphical representation of the forging pressure against distance for both finite element and the exact solution for the forging process is plotted. As shown in Figure 2 the comparison between the plot of pressure and the distance obtained from the numerical analysis (i.e., using Bubnov-Galerkin weighted-residual method) and the analytical method (i.e., exact solution method) shows close similarity.

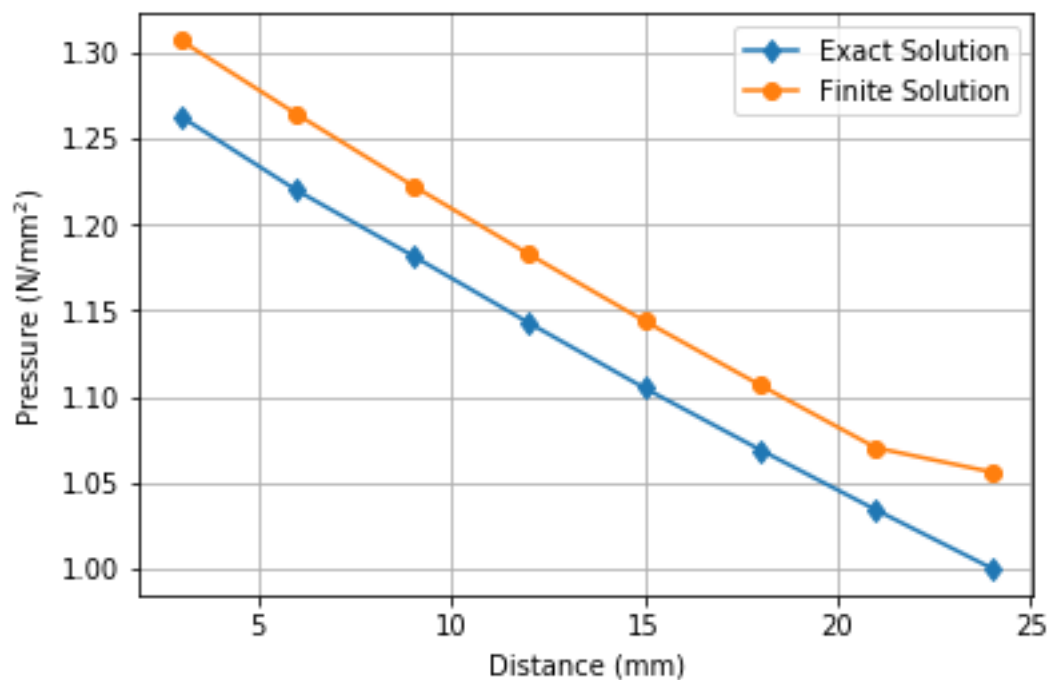


Figure 2: Graphical comparison of the Exact solution and the finite element method for example 1.

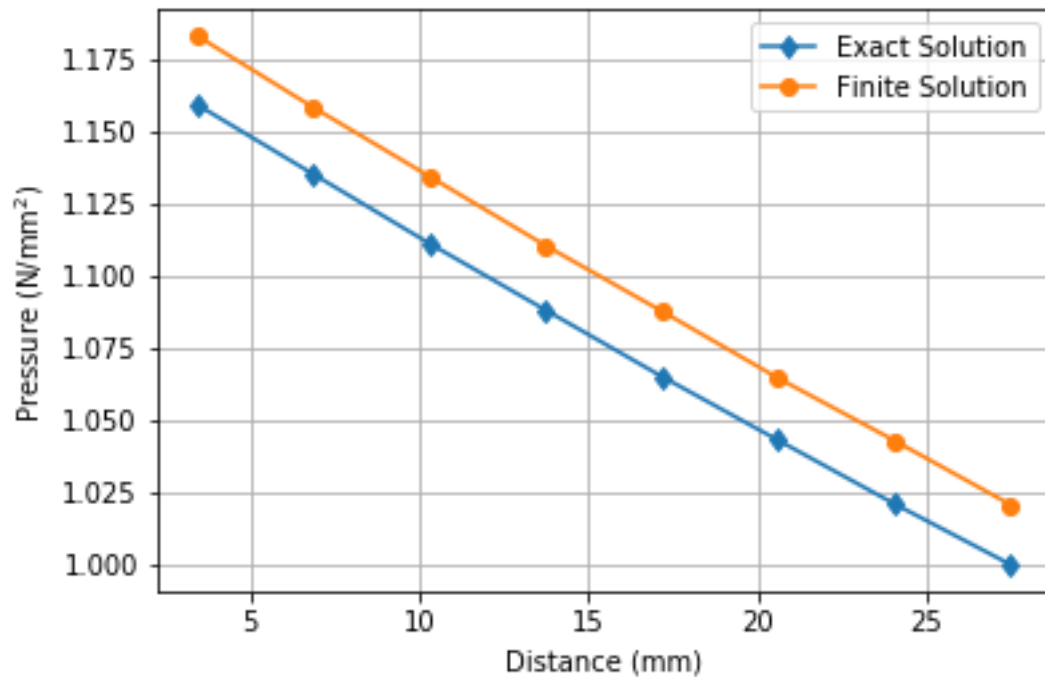


Figure 3: Graphical comparison of the Exact solution and the finite element method for example 2.

3.2: DISCUSSION OF RESULTS

To illustrate the use and accuracy of the FEM, two examples were considered and their solution obtained for both exact and FEM. The percentage differences between the compared solution of the finite element method and exact method were shown in Table 3 and Table 4 for various radius of each element of the metal forming process (forging). The results shows the ability of finite element method to be applicable in the accurate determination of pressures, temperatures and other environmental variants in the material or equipment being analyzed in metal forming process.

In Table 3, it is shown that the maximum and minimum percentage difference ranges from 0.03 % and 0.05 % , and Table 4, ranges from 0.01 % and 0.02 % between the finite element solution and the exact solution. Therefore, the overall maximum percentage difference of 0.05 % and a minimum difference of 0.01% depicts that both solutions obtained through FEM and Exact method is very accurate. In addition, Figure 2 and Figure 3 clearly shows that there is a difference in the flow of behaviour for both method at a uniform pressure, and when the pressure decreases.

4.0: CONCLUSION AND RECOMMENDATION

In this work, it can be concluded that the weighted residual finite element method can be used to predict pressures accurately when set up in a metal forming process (i.e., forging). The analytical model developed in this paper is slightly in concurrence and accurate to the finite element method result obtained as shown in the result. Furthermore, this present work can be expanded to other metal forming processes with a two- or three-dimensional geometry to establish more theoretical findings.

Furthermore, this work can be expanded to other metal forming processes with a two or more-dimensional geometry to establish more theoretical findings. The results also shows the ability of finite element method to be applicable in the accurate determination of pressures as stated in this study, it is recommended for future work to be carried out on determination of forces, strains, temperatures and other environmental variants in any metal forming material or engineering analysis using the numerical and experimental methods. Also, a numerical simulation of the microstructural behavior during forging process can be studied using various FEA software.

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